#### Calculation of Physical Properties of Seawater

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### 1. Introduction

Standard level listings of physical properties of seawater appear in cruise reports as part of the station data summaries. This section contains a description of the methods for making these calculations. The physical properties of seawater algorithms described follow Fofonoff and Millard (1983). Those algorithms were developed using the International Practical Temperature Scale of 1968 (IPTS-68) while the International Temperature Scale of 1990 is recommended for reporting observations. An error estimate associated with using ITS-90 with these algorithms is provided together with a linear temperature scale transformation when required. The variable descriptions are grouped into categories of: measured or state variables; physical properties at standard levels; properties integrated to standard levels; and vertical gradients of properties at standard levels. A table at the end of this section contains a list of commonly used oceanographic units together with MKS and SI unit equivalent for the oceanographic variables discussed.

## 2. Measured and State Variables

The CTD has sensors to measure conductivity C, temperature t, and pressure p versus time. The regular calibration of these sensors against certified transfer standards is essential for accurate measurements. Calibration methods are discussed in Chapter 1 of this document (Saunders  $et\ al$ , 1991) and by SCOR WG 51 (Unesco,1988). This section contains recommendations for units and estimates for errors arising if the temperature scale conversion is ignored.

**Pressure:** The pressure not only indicates the level at which the observation takes place but is a state variable which enters into the calculation of most physical properties of seawater. The normal reporting unit of pressure is decibars (dbars) with 1 dbar equivalent to  $10^4$  pascals (kg m<sup>-1</sup>s<sup>-2</sup>). The oceanographic convention is to report pressure as applied, or gauge pressure, rather than total pressure, *i.e.*, p = 0 at the ocean surface.

Temperature: Temperature has been reported over the past 22 years on the International Practical Temperature Scale of 1968 (IPTS-68). All of the algorithms for calculating physical properties of seawater have been developed using IPTS-68. The Comite International des Poids et Mesures has recommended the adoption of the International Temperature Scale of 1990 (ITS-90). ITS-90 introduces the melting point of gallium (29.7646 °C) as a reference point in the new temperature scale. The triple-point of water (.0100 °C)

remains unchanged on ITS-90 and the following linear transformation

$$t_{68} = 1.00024 \cdot t_{90} \tag{1}$$

is accurate within 0.5 m°C for conversion between IPTS-68 and ITS-90 over the oceanographic temperature range (Saunders, et al 1991).

Conductivity Ratio: Most CTDs are calibrated and report electrical conductivity of seawater in milliSiemens/cm (mS/cm). However, the conductivity ratio, R, is required for the calculation of salinity. The conductivity ratio is the *in situ* conductivity divided by the conductivity at a salinity of 35 (PSS-78), temperature 15 °C (IPTS-68), and standard atmosphere  $p_a = 10.13250$  dbars.

$$R = C(S, t, p)/C(35, 15, 0)$$
 (2)

A frequently used value of C(35, 15, 0) is 42.914 mS/cm for temperature measured on IPTS-68. If conductivities are recorded, the value of C(35, 15, 0) must also be recorded to ensure accurate conversion to conductivity ratios.

Salinity: Salinity is calculated from conductivity ratio, temperature, and pressure using the Practical Salinity Scale of 1978 (PSS-78), Lewis (1980), Unesco (1981). A Fortran algorithm for computing PSS-78 salinity from pressure, temperature and electrical conductivity ratio is found in Fofonoff and Millard (1983).

The range of validity of PSS-78 is -2 < t < 35 °C and 2 < S < 42. The pressure range 0 dbars.

The ITS-90 has a significant effect on the calculation of salinity as the PSS-78 algorithm was developed on the IPTS-68 temperature scale. The following tables of salinity differences at two conductivity ratios indicate the magnitude of the error when conversion of ITS-90 to IPTS-68 is ignored. Salinity values for the two conductivity ratios at pressures of 0, 5,000 and 10,000 dbars can be obtained from the salinity tables in Unesco (1983, p. 13).

Salinity Difference Table

$$\Delta S = (S_{t_{90}} - S_{t_{68}}) \quad R = 1.3981451$$

Pressure	Temperature °C ITS-90				
dbars	0.0	10.0	20.0	30.0	40.0
0 2500 5000 7500 10000	0.0000 0.0000 0.0000 0.0000 0.0000	0.0040 0.0038 0.0036 0.0035 0.0034	0.0052 0.0050 0.0048 0.0047 0.0046	0.0054 0.0052 0.0050 0.0049 0.0048	0.0052 0.0050 0.0049 0.0048 0.0047

Salinity Difference Table

$$\Delta S = (S_{t_{90}} - S_{t_{68}}) \qquad R = 1.165120$$

Pressure	Temperature °C ITS-90				
dbars	0.0	10.0	20.0	30.0	40.0
0 2500 5000 7500 10000	0.0000 0.0000 0.0000 0.0000 0.0000	0.0032 0.0030 0.0029 0.0028 0.0027	0.0042 0.0040 0.0039 0.0038 0.0037	0.0044 0.0042 0.0041 0.0040 0.0039	0.0042 0.0041 0.0039 0.0039 0.0038

Oxygen: The method for converting oxygen from polarographic sensor data is given by Owens and Millard (1985). The oxygen saturation value is required and the method of Weiss (1970) has generally been used. The adoption of the Benson and Krause (1984) oxygen saturation formula is recommended by the Joint Panel of Oceanographic Tables and Standards (UNESCO, 1986) as it incorporates improved oxygen solubility measurements. The calibration technique is described elsewhere in this manual. The ITS-90 has a negligible effect on the oxygen calculation. The conversion of oxygen content from milliliter/liter (ml/l) to  $\mu$ mol/kg is made by multiplying the oxygen content in ml/l by the molar volume of oxygen (44.660  $\mu$ mol/l) and dividing by the potential density expressed in kg/l (see Culberson, 1991). Note that 1 kg/l = 1000 kg/m<sup>3</sup>.

# 3. Physical Properties of Seawater at Standard Pressure Levels

#### Adiabatic Lapse Rate and Potential Temperature

The adiabatic lapse rate , (S,t,p) (°C/dbar) is defined as the change of temperature per unit pressure for an adiabatic change of pressure of an element of seawater. It is assumed that no heat or salt is exchanged with the surroundings so that the pressure changes are both adiabatic and isentropic. From thermodynamical considerations, the adiabatic lapse rate, , a function of pressure, temperature, and salinity can be expressed as

$$, = T \frac{\partial V}{\partial t} / C_p \tag{3}$$

where  $T=t+273.15~\mathrm{K}$  is absolute temperature (Kelvin),  $\frac{\partial V}{\partial t}$  (m<sup>3</sup>/(kg °C)) is the thermal expansion and  $C_p$  (J/(kg °C)) specific heat of seawater at constant pressure.

An algorithm for adiabatic lapse rate has been calculated from the equation of state and specific heat following Bryden (1973). Bryden's formulas are selected because of their agreement with values obtained from the 1980 international equation of state of seawater (EOS-80).

Potential temperature  $\theta(S,t,p,p_r)$  has been defined classically as the temperature an element of seawater would have if raised adiabatically with no change of salinity to atmospheric pressure. More generally, the potential temperature can be defined as the temperature resulting from an adiabatic displacement to a reference pressure  $p_r$  that may be greater or less than the initial pressure p. The potential temperature  $\theta$  is computed from the adiabatic lapse rate p, p,

$$\theta(S, t, p, p_r) = t + \int_p^{p_r} (S, t', p') dp'$$
 (4)

by integration along an adiabat, i.e.,  $\left(\frac{\partial t'}{\partial p'}\right)_A =$ , '. The potential temperature can be evaluated by using an empirical formula or by numerical integration of the defining equation. The numerical integration by the Runge-Kutta method is provided as a Fortran routine by Fofonoff and Millard (1983). The potential temperature  $\theta(S,T,p,p_r)$  at reference pressure  $p_r$  is computed with sufficient precision using a 4-th order Runge-Kutta integration algorithm (Fofonoff, 1977).

The effects of ITS-90 on the calculation of potential temperature arise from the slight shift in the adiabat followed in moving between pressure levels. An extreme over-estimate of between 0.3 and 0.4 m°C is found in the potential temperature when the calculation is made on the ITS-90 without conversion first to IPTS-68 then back to ITS-90.

The 1980 international equation of state of seawater (EOS-80), using standard seawater diluted with pure water or concentrated by evaporation, has been determined for use with the practical salinity scale (Millero et al., 1980; Millero and Poisson, 1981). This equation is more precise than the earlier equations (Knudsen et al., 1902; Ekman, 1908; Cox et al., 1970) and covers a wider range of temperature and pressure. Data reports describing the details of the fitting procedure are available (Millero, Chen, Bradshaw and Schleicher, Unesco Report No. 38, (1981); Millero and Poisson, Unesco Report No. 38, (1981)).

The density,  $\rho$ , in kg/m<sup>3</sup>, of seawater as a function of practical salinity S, temperature, t,  ${}^{\circ}$ C, and applied or gauge pressure, p, in decibars, is given by:

$$\rho(S, t, p) = \rho(S, t, 0)/[1 - p/K(S, t, p)]$$
(5)

where K(S, t, p) is the secant bulk modulus. The specific volume  $(V = 1/\rho)$ , in m<sup>3</sup>/kg of seawater can be obtained from

$$V(S, t, p) = V(S, t, 0) \cdot [1 - p/K(S, t, p)]$$
(6)

Because the maximum variation of density and specific volume of seawater is less than 7% over the oceanic ranges of variables, numerical precision is lost by retaining the full numerical magnitude. A 32-bit single precision floating point number has about 7 significant decimal digits. While such precision exceeds the accuracy of the measurements and is adequate for most purposes where the full value is required, it is insufficiently accurate for taking differences for Brunt-Väisälä frequency calculations. An improvement of two orders of magnitude in the oceanic range is obtained by computing the specific volume anomaly  $\delta$  defined by:

$$\delta(S, t, p) = V(S, t, p) - V(35, 0, p) \tag{7}$$

and reported in units of  $10^{-8}$  m<sup>3</sup>/kg. V(35,0,p) is the specific volume of a "standard" ocean.

For many oceanographic applications, such as dynamic height and potential energy anomaly calculations, the specific volume (or steric) anomaly,  $\delta$ , provides a more convenient measure of the baroclinic structure in the ocean. For computational convenience, V(35,0,p) has been obtained from the EOS-80 by evaluating the coefficients for a salinity of 35 (PSS-78) and temperature of 0 °C IPTS-68 in the form

$$V(35,0,p) = V(35,0,0) \cdot [1 - p/K(35,0,p)] \tag{8}$$

Coefficients are given in Fofonoff and Millard (1983).

The density anomaly,  $\gamma$  in kg/m<sup>3</sup>, is defined by

$$\gamma(S, t, p) = 1/V(S, t, p) - 1000 \text{ kg/m}^3$$
(9)

The potential density anomaly at an arbitrary reference pressure level,  $p_r$ , can be obtained by substituting in the density anomaly formula the reference pressure,  $p_r$ , and replacing the  $in\ situ$  temperature with the potential temperature at the reference pressure, i.e.,

$$\gamma_{p_r} = \gamma(S, \theta(S, t, p, p_r), p_r) \tag{10}$$

The ITS-90 has the following effect on the calculation of density anomaly for S=35 and 40. A similar effect occurs at other salinities.

#### Density Anomaly Difference Table

$$\Delta \gamma = (\gamma_{t_{90}} - \gamma_{t_{68}}), \, \text{kg/m}^3 \, S = 35$$

Pressure	Temperature °C ITS-90				
dbars	0.0	10.0	20.0	30.0	40.0
0 2500 5000 7500 10000	0.0000 0.0000 0.0000 0.0000 0.0000	0.0004 0.0005 0.0006 0.0007 0.0008	0.0013 0.0014 0.0015 0.0017 0.0018	0.0025 0.0026 0.0027 0.0028 0.0029	0.0039 0.0040 0.0041 0.0042 0.0042

#### Density Anomaly Difference Table

$$\Delta \gamma = (\gamma_{t_{90}} - \gamma_{t_{68}}), \, \text{kg/m}^3 \, S = 40$$

Pressure	Temperature °C ITS-90				
$_{ m dbars}$	0.0	10.0	20.0	30.0	40.0
0 2500 5000 10000	0.0000 0.0000 0.0000 0.0000 0.0000	0.0004 0.0006 0.0006 0.0007 0.0008	0.0013 0.0014 0.0016 0.0017 0.0018	0.0025 0.0026 0.0027 0.0029 0.0030	0.0039 0.0040 0.0041 0.0042 0.0043

The density anomaly differences are largest at high temperatures. Although the differences between ITS-90 and ITS-68 are small at low temperatures and do not affect density anomalies significantly for the deep ocean ranges, conversion to the IPTS-68 scale is recommended for computation of density at all levels.

#### Pressure to Depth Conversion

Saunders and Fofonoff (1976) developed an accurate formula for pressure to depth conversion using the hydrostatic equation and the Knudsen-Ekman equation of state. The formula included variation of gravity with latitude and depth. For simplicity and consistency, a new formula (in SI units) based on EOS-80 has been developed by Fofonoff and Millard (1983) that is accurate to 0.1 m over the pressure range 0–10,000 dbars. It should be noted that Saunders (1981) has developed a simpler but less precise quadratic formula based on EOS-80.

Following Saunders and Fofonoff, the hydrostatic equation is integrated in the form

$$\int_0^z g dz = [g_0(\phi) + 1/2\gamma_g z]z = \int_0^p V dp = \int_0^p V(35, 0, p) dp + \Delta D$$
 (11)

where  $g_0(\phi)$  is gravity at the ocean surface, a function of latitude and  $\gamma_g$  is the mean vertical gradient of gravity in the ocean, V, is specific volume and  $\Delta D$ , the geopotential anomaly, is discussed later.

A depth algorithm involving an approximation to the integration is given by Fofonoff and Millard (1983). The effects of ITS-90, through the small geopotential anomaly correction to depth, are negligible.

# 4. Properties Integrated to Standard Pressure Levels

#### Geopotential (Dynamic Height) Anomaly and Potential Energy Anomaly

The geopotential (dynamic height) and potential energy anomalies are used in the calculation of geostrophic velocity and mass transport respectively.

If the field of mass is known, the internal field of pressure can be determined from the equation of static equilibrium (hydrostatic equation)

$$dD = Vdp (12)$$

Following Sverdrup, Johnson and Fleming (1942), the geopotential (dynamic height) anomaly is

$$D = \int_{p_0}^{p_n} V dp = D_0 + \int_{p_0}^{p_n} \delta dp \tag{13}$$

$$\Delta D = \int_{p_0}^{p_n} \delta dp \tag{14}$$

Using the trapezoidal rule of integration it follows that

$$\Delta D = \frac{\delta_0 + \delta_1}{2}(p_1 - p_0) + \frac{\delta_1 + \delta_2}{2}(p_2 - p_1) + \dots + \frac{\delta_{n-1} + \delta_n}{2}(p_n - p_{n-1})$$
 (15)

where  $0, 1, 2, \ldots, n$  are the individual 2-dbar observations for computer-derived values.

The units for geopotential anomaly  $(10 \text{ J/m}^2)$  are chosen so that geopotential differences between stations approximate numerically height differences of pressure surfaces in meters.

#### Potential Energy Anomaly

Following Fofonoff (1962), the potential energy anomaly,  $\chi$ , is

$$\chi = \frac{1}{g(\phi)} \int_{p_0}^{p_n} p \delta dp \tag{16}$$

The potential energy anomaly,  $\chi$ , is computed using the trapezoidal rule in the form

$$\chi = \frac{1}{g} \left[ \frac{\delta_0 p_0 + \delta_1 p_1}{2} (p_1 - p_0) + \frac{\delta_1 p_1 + \delta_2 p_2}{2} (p_2 - p_1) + \dots + \frac{\delta_{n-1} p_{n-1} + \delta_n p_n}{2} (p_n - p_{n-1}) \right]$$
(17)

where  $0, 1, 2, \ldots, n$  are the individual 2-dbar observations for computer-derived values.

The units for potential energy anomaly are chosen so that differences from station to station at a pressure level p correspond approximately to the geostrophic mass transport in Sverdrups ( $10^6 \text{ m}^3/\text{s}$  or  $10^9 \text{ kg/s}$ ) relative to that level. A accurate estimate is obtained by dividing the difference by  $10^4 f$ , where f is the local Coriolis parameter.

A Fortran routine for calculating the geopotential and potential energy anomalies using the trapezoidal integration method is provided in Unesco (1991). The error introduced by ITS-90 to dynamic height and potential energy anomaly comes into the calculation through the specific volume anomaly. The effect of using ITS-90 on geopotential anomaly was tested for a western Pacific profile off the Philippines with a surface temperature of 28 °C. The geopotential anomaly was underestimated by  $0.01 \, \mathrm{m^2/s^2}$  on integration to both 1000 and 4000 dbars, while the potential energy anomaly was underestimated by  $10^4 \, \mathrm{J/m^2}$  at both the pressure levels. Except perhaps for the Mediterranean and Red sea, this is the maximum expected error for the world's oceans. The differences are negligible.

#### Sound Speed in Seawater

According to the theory of sound-wave propagation (Lamb, 1932), the speed of sound, c, is given by the formula

$$c^2 = (\partial p/\partial \rho)_A \tag{18}$$

where  $(\partial p/\partial \rho)_A$  is the reciprocal of the adiabatic density gradient in a compressible fluid.

Sound speed has been measured for samples of standard seawater, diluted with pure water or concentrated by evaporation, by Chen and Millero (1977). The formula developed from these measurements is consistent with the new salinity scale and is in better agreement with values computed from EOS-80 than the formulas given by Wilson (1960) and Del Grosso and Mader (1972).

The Fortran code for the speed of sound in seawater, following Chen and Millero (1977), is found in Fofonoff and Millard (1983).

The average speed of sound over the water column is computed from the depth integrated travel time  $\overline{T}$  as follows:

$$\overline{T} = \int_0^z \frac{1}{c(z)} dz = \int_0^{p(z)} \frac{1}{\rho(p)g(\phi, p)c(p)} dp$$
 (19)

$$\overline{c} = 1/\overline{T} \text{ m/s}$$
 (20)

The effect of ITS-90 on the speed of sound is negligible. An over-estimate of sound speed of 0.01 m/s occurs at 30 °C when using the ITS-90 temperature instead of IPTS-68.

# 5. Gradient Properties at Standard Pressure Levels

#### Brunt-Väisälä Frequency

The Brunt-Väisälä frequency, N, is the natural frequency of oscillation of a water parcel displaced adiabatically from its rest position. It determines the high frequency limit for internal waves (Phillips, 1966; Gill, 1982). In addition, the stability, E, which is proportional to  $N^2$ , is required to estimate the vertical vortex stretching contribution to potential vorticity (McDowell  $et\ al.$ , 1982).

The calculation of the Brunt-Väisälä frequency involves the estimation of the differences in the vertical gradient of in situ density and potential density. Three methods are now commonly used in the oceanographic community to estimate these derivatives. They are: the Fofonoff adiabatic, steric leveling method (Bray and Fofonoff, 1981); the Hesselberg-Sverdrup method (Hesselberg and Sverdrup, 1915), which uses vertical gradients of temperature and salinity together with local expansion coefficients; and the adiabatic correction (or speed of sound) method (Sverdrup et al., 1942, Phillips, 1966). The adiabatic, steric anomaly leveling method is recommended for calculating Brunt-Väisälä frequency (Millard et al., 1990).

$$N^2 = -g^2 \frac{\Delta \rho}{\Delta p} \tag{21}$$

where  $\Delta \rho$  and  $\Delta p$  are finite differences of in situ potential density and pressure over a vertical interval. The stability parameter, E is defined as (Hesselberg, 1918)

$$E = \frac{1}{\rho} \left( \frac{\Delta \rho}{\Delta z} \right), \tag{22}$$

which gives

$$E = \frac{N^2}{q} (23)$$

If one expands  $\Delta \rho$  as a Taylor series in pressure about  $p_0$ , using  $\theta(S_0, t_0, p_0) = t_0$ , one obtains

$$\Delta \rho = \left(\frac{\partial \rho}{\partial p}(S_0, \theta(S_0, t_0, p_0, p), p)|_{p_0} - \frac{\partial \rho}{\partial p}(S, t, p)|_{p_0}\right) dp + O(dp^2), \tag{24}$$

where  $|_{p_0}$  means evaluated at  $p_0$ . Ignoring  $O(dp^2)$  terms, substituting into (21) and letting  $dp \to 0$ , gives

$$N^{2} = g^{2} \left( \frac{\partial \rho}{\partial p} (S, t, p) - \frac{\partial \rho}{\partial p} (S_{0}, \theta(S_{0}, t_{0}, p_{0}, p), p) \right), \tag{25}$$

where the right-hand side is the difference between the vertical gradients of in situ density and of potential density.

In the Fofonoff adiabatic leveling methods (Bray and Fofonoff, 1981) the vertical finite difference in equation (21) is replaced by the average of moving two parcels of water, one from above and one from below the reference pressure, p. This formulation was also carried out using the specific volume  $V = 1/\rho$ , so that

$$d\rho = -\frac{1}{V^2}dV. (26)$$

To increase numerical resolution, one uses the steric anomaly,  $\delta$ , defined as

$$V(S, t, p) = \delta(S, t, p) + V(35, 0, p)$$
(27)

Substituting into equation (25), using (26) and (27), and averaging the contributions from the two parcels, gives

$$N^{2} = \frac{g^{2}}{2 V(S, t, p)^{2}} \left[ \frac{\delta(S_{1}, \theta(S_{1}, t_{1}, p_{1}, p), p) - \delta(S, t, p)}{(p_{1} - p)} + \frac{\delta(S_{2}, \theta(S_{2}, t_{2}, p_{2}, p), p) - \delta(S, t, p)}{(p_{2} - p)} \right]$$
(28)

where  $(p_1 + p_2)/2 = p$ , and  $p_1$  and  $p_2$  are the two equilibrium pressures for the two water parcels. Rearrangement gives

$$N^{2} = \frac{g^{2}}{V(S, t, p)^{2}} \left[ \frac{\delta(S_{2}, \theta(S_{2}, t_{2}, p_{2}, p), p) - \delta(S_{1}, \theta(S_{1}, t_{1}, p_{1}, p), p)}{(p_{2} - p_{1})} \right]$$
(29)

When this method is extended to estimate the vertical derivatives over a pressure interval using a least-squares fit, the term in brackets can be replaced by the least-squares estimate of the slope, provided the potential steric anomalies are referenced to the average pressure for the interval. A Fortran routine for calculating the Brunt-Väisälä frequency using the Fofonoff adiabatic leveling method is provided in Unesco (1991).

# 6. Miscellaneous Properties

#### Gravity

Gravity,  $g(\phi, p)$  in m/s², corrected for latitude,  $\phi$ , and pressure, p, dbars is given by

$$g(\phi, p) = 9.7803185 \cdot (1.0 + 5.278895 \cdot 10^{-3} \cdot \sin^2 \phi + 2.3462 \cdot 10^{-5} \cdot \sin^4 \phi) + \frac{1}{2} \gamma_g' p (30)$$

where  $\gamma_g'=2.184\cdot 10^{-6}$  is the mean vertical gradient of gravity versus pressure (Anon, 1970; Fofonoff and Millard,1983).

#### Coriolis Parameter

The Coriolis parameter f as a function of latitude is given by

$$f = 2\Omega_e \sin \phi = 1.458432 \cdot 10^{-4} \cdot \sin \phi \text{ radians/s}$$
 (31)

where  $\Omega_e$  is the earth's angular velocity of rotation based on the sidereal day in seconds.

Dimensions and Units of Oceanographic Variables						
Variable		Oceanographic Units	SI/MKS units			
Pressure, $p$		dbars 1000.0	$ m kg~m^{-1}~s^{-2}$ $10^7~Pascals$			
Temperature, $t$	ITS-90	$^{\circ}\mathrm{C}$ $4.726$	°C 4.726			
Potential temperature, $\theta$		$^{\circ}\mathrm{C}$ $4.645$	$^{\circ}\mathrm{C}$ $4.645$			
Salinity, S	PSS-78	34.551	34.551			
$Oxygen,^* O_2$		$\frac{\mathrm{ml}/\mathrm{l}}{1.18}$	$ m \mu mol/kg \ 51.413$			
Density, $\rho$	EOS-80	$ m kg/m^3 \ 1031.9436$	$ m kg/m^3 \ 1031.9436$			
Density anomaly, $\gamma$	EOS-80	$\frac{\rm kg/m^3}{31.9436}$	$\frac{\rm kg/m^3}{31.9436}$			
Specific volume, $V$	EOS-80	${ m m}^3/{ m kg} \ 0.000978297$	${ m m}^3/{ m kg} \ 0.000978297$			
Specific volume anomaly, $\delta$	EOS-80	$10^{-8} \text{ m}^3/\text{kg}$ $82.078$	$rac{{ m m}^3/{ m kg}}{82.078 \cdot 10^{-8}}$			
Sound speed, $c$		$\frac{\mathrm{m/s}}{1485.5}$	m m/s $1485.5$			
Dynamic height, $\Delta$ $D$		$10 \text{ m}^2/\text{s}^2 \\ 1.747$	m J/kg 17.47			
Potential energy anomaly, $\chi$		$10^5 \ \mathrm{J/m^2} \ 73.5$	${\rm J/m^2} \\ 73.5 \cdot 10^5$			
Brunt-Väisälä frequency, $N$		$ m cycles/hour \ 1.52$	$\begin{array}{c} \rm radians/s \\ 2.65 \cdot 10^{-3} \end{array}$			
Depth, $z$		$^{\rm m}_{992}$	m 992			

 $<sup>^*</sup>$  for a seawater density of 1.025 x  $10^3~{\rm kg/m^3}$ 

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